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SOLVING MINIMAL COVERING LOCATION PROBLEMS WITH SINGLE AND MULTIPLE NODE COVERAGE

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Abstract

Location science represents a very attractiveresearch field in combinatorial optimization and it is in expansion in last five decades. The main objective of location problems is determining the best position for facilities in a given set of nodes.Location science includes techniques for modelling problemsand methods for solving them. This paper presents results of solving two types of minimal covering location problems, with single and multiple node coverage, by using CPLEX optimizer and Particle Swarm Optimization method.

Keywords:minimal covering location problem, single and multiple coverage, particle swarm optimization

1. Introduction

Interest in modelling and solving covering location problems have grown in the last fifty years. Many papers from this field have been published, but the main focus was on problems with maximal coverage, such as Location set coverage problem (LSCP) [13] and Maximal covering location problem (MCLP) [2]. On the other hand, problems with minimal coverage have not been studied much in the past. The most important problems from this class are Anti-coverage location problem (ACLP) introduced by Moon and Chaudhry in [10] and Minimum covering location problem with distance constraint (MCLPDC) introduced by Berman and Huang in [2]. This paper presents results of tests for solving two types of Minimal covering location problem presented in [6] – Minimal covering location problem with single node coverage (MinCLP-SC) and Minimal covering location problem with multiple node coverage (MinCLP-MC). Both problems have been solved on generated instances with 100, 200, 300, 400 and 500 nodes. Firstly, we tried to solve generated instances with CPLEX optimizer and, if CPLEX did not find a solution, instances were solved by using Particle Swarm Optimization (PSO) method.

This paper is organized as follows. In Section 2, mathematical models of MinCLP-SC and MinCLP-MC are presented. Section 3 gives a brief description of CPLEX optimizer and PSO method. In Section 4, test results are given, with discussion and graphical representations of results of two instances. At the end, in the Section 5, we conclude this paper and present plans for future research in this field.

2. Minimal covering location problem

As mentioned before, the minimal covering location problem (MinCLP) has not been studied much in the past and the most exhaustive study related to this area could be found in [2]. Some authors have studied similar problems with minimal coverage, but with different formulations and names, such as Expropriation location problem(ELP)[1,11]. For more information and models about locating undesirable facilities, we refer on [2,9].

The objective of Minimal covering location problem is to locate fixed number of facilities in a given set of nodes, in such way that the total coverage is minimal. Three main parameters of MinCLP are the number of nodes (dimension of the problem), the number of facilities to be located and the radius of coverage. MinCLP has an application in problems of locating undesirable objects, such as pollutants, power and nuclear plants, jails etc.

The aim of this study is solving two mathematical models of MinCLP defined in [6]. Before presenting these models, it is necessary to define the following parameters:

- I – set of locations (indexed by i),
- J – set of eligible facility sites (indexed by j),
- S – radius of coverage,
- d_{ij} – distance from location i to location j,
- $x_j = \begin{cases} 1, \text{if facility is located at location } j, \\ 0, \text{otherwise}, \end{cases}$
- P – number of facilities,
- $N_i = \{j \in J | d_{ij} \leq S\}$ – set of all facilities j which cover location i.

Mathematical model of MinCLP with single coverage (MinCLP-SC) is the following:

$$\text{minimize} \quad g = \sum_{i \in I} y_i \qquad (1)$$

$$\text{subject to} \quad \sum_{j \in J} x_j = P \qquad (2)$$

$$\sum_{j \in N_i} x_j \leq y_i, \forall i \in I \qquad (3)$$

$$x_j \in \{0,1\}, \forall j \in J \qquad (4)$$

$$y_i \in \{0,1\}, \forall i \in I \qquad (5)$$

In this model, the aim is to minimize objective function (1) with following conditions: the number of facilities must be equal to P (2), coverage of each node $y_i$is limited by the condition (3) and conditions (4) and (5) define decision variables x_i and y_i. The condition (2), together with condition (5) determines the main property of this model – each node can be covered by at most one facility. This condition is useful for problems of this type – there are no nodes covered by multiple undesirable facilities and that is a desirable property. But, in many cases, this condition narrows solution space and leads to the impossibility of problem solving.

To overcome this deficiency, it is necessary to limit sum from condition (3) with number one. Finally, to prevent that all facilities have been located in the same region, distance constraint is introduces:

- d – minimal distance between facilities.

Mathematical model of MinCLP with multiple node coverage (MinCLP-MC) is:

$$\text{minimize} \quad g = \sum_{i \in I} y_i \qquad (6)$$

$$\text{subject to} \quad \sum_{j \in J} x_j = P \qquad (7)$$

$$\min(1, \sum_{j \in N_i} x_j) \leq y_i, \forall i \in I \qquad (8)$$

$$d_{ij} \geq d, \forall j_1, j_2 \in J, j_1 < j_2 \wedge x_{j_1} \cdot x_{j_2} = 1 \qquad (9)$$

$$x_j \in \{0,1\}, \forall j \in J \qquad (10)$$

$$y_i \in \{0,1\}, \forall i \in I \qquad (11)$$

This model is similar to the previous, with two changes: sum in (8) is limited by number one and condition (9) satisfies minimal distance constraint.

3. IBM CPLEX optimizer and Particle Swarm Optimization method

As described in Section 1, in this study we tried to solve generated instances by exact algorithm of CPLEX optimizer and if it was unsuccessful, therefore, we solved them by Particle Swarm Optimization method.

IBM ILOG CPLEX optimizer is optimization software which provides different algorithms for exact solving problems of linear, mixed integer, quadratic and quadratically constrained programming [5]. Theoretically, CPLEX optimizer could solve problems with millions of constraints and variables, but in practice, unacceptable time and/or memory is required for solving some problems.

Particle Swarm Optimization method is ametaheuristic inspired by social behaviour of particles in swarms, such as birds or fishes in flocks. It has been introduced by Kenedy and Eberhart in 1995. In this paper, we used the adaptation of PSO for solving problems from discreet binary space - Discreet PSO (DPSO). A brief overview of PSO, together with its adaptation to solving MinCLP will be presented here, but detailed theoretical background and practical approaches could be found in [8].

The main idea of PSO is creating a swarm of several particles, which flies through solution space. Each particle is built by two vectors: the position vector $\boldsymbol{x_i}$from d-dimensional binary space and the velocity vector $\boldsymbol{v_i}$ from$d$-dimensional continuous space**.** Particle updates its position by using the information about its best position so far and the best position of particles from its neighbourhood. Velocity vector represents the probability that the j-th binary value from position vector$\boldsymbol{x_i}$ obtains a value 1. By the definition, $\boldsymbol{x_{ij}}$ gets value 1 if randomly generated value is less than:

$$\frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{e^{-v_{ij}}}}$$

Velocity vector in k-th iteration is updated by formula:

$$\boldsymbol{v_i^k = v_i^{k-1} + c_2\xi_1(b_i - x_i^{k-1}) + c_3\xi_2(c_i - x_i^{k-1})} \; (\mathbf{12})$$

where $\boldsymbol{\xi_1}$ and $\boldsymbol{\xi_2}$ are randomly generated values from [0,1];$\boldsymbol{b_i}$ particle's best position so far and$\boldsymbol{c_i}$ the best position of particles from neighbourhood; $\boldsymbol{c_2}$ and $\boldsymbol{c_3}$ are known as cognitive and social factor and they represent particle's movement dynamic to its best position and to the best position from the neighbourhood. For preventing the velocity vector from taking too small or too large values, its values are limited with some interval, usually -6 < v_{ij} < 6.

4. Computational tests

Both problems are solved on generated instances. Instances are generated by using procedure presented in [8] - locations are randomly set in 30x30 grid. Problems are solved on instances up to 500 nodes with parameters: coverage radius S=2,3,4,5 number of facilities P=10,15,20 and distance constraint in MinCLP-MC is d=4.

Both algorithms (based on CPLEX and PSO) are coded in Visual C#.NET 2010, all tests were running on the computer with Intel Core i7-800 2,8GHz processor with 8GB RAM memory and Windows 7 Professional operating system. In the first algorithm, IBM CPLEX optimizer v12.1 teaching edition has been used.

4.1. Solving MinCLP-SC

Test results of using CPLEX optimizer for solving MinCLP-SC are presented in Table 1. Because of the lack of space, only instances of 500 nodes are presented, but results for all instances are available at http://personal.ffuis.edu.ba/ddrakulic/phd/tests.

Table 1: Solving MinCLP-SC by using CPLEX

n	P	S	Solution	Time(ms)
500	10	2	21	66
500	10	3	60	85
500	10	4	121	165
500	10	5	200	234
500	15	2	36	56
500	15	3	99	92
500	15	4	210	117
500	15	5	-	-
500	20	2	21	73
500	20	3	145	86
500	20	4	337	183
500	20	5	-	-

Table 1 shows that CPLEX has successfully solved all given instances for a short time. It also illustrates the disadvantage of this model – instances with 15 nodes, 20 facilities and coverage radius 5 do not have a solution. Figure 1 illustrates the graphical representation of solution for 300 nodes, 15 facilities and radius of coverage 4.

4.1. Solving MinCLP-MC

As mentioned above, MinCLP-MC has been solved on same instances with same parameters, with additional distance constraint parameter $\boldsymbol{d}$=4. CPLEX successfully solved only instances of up to 400 nodes. Table 2 presents results of tests for solving instances of MinCLP-MC with 400 nodes by using CPLEX.

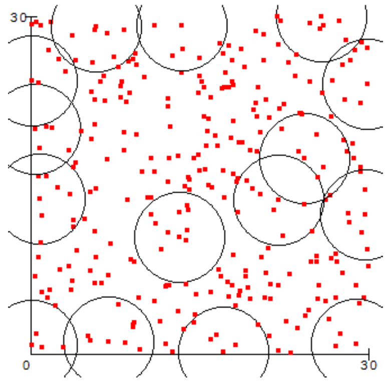

Fig. 1: Graphical representation of MinCLP-SC solution

Table 2: Solving MinCLP-MC by using CPLEX

n	P	S	Solution	Time (ms)
400	10	2	17	68273
400	10	3	43	70872
400	10	4	74	94171
400	10	5	82	130755
400	15	2	27	69506
400	15	3	73	74153
400	15	4	112	99328
400	15	5	124	141916
400	20	2	42	72083
400	20	3	104	75807
400	20	4	150	94597
400	20	5	167	426092

Table 2 illustrates that MinCLP-SC is much harder to solve than previous problem. Actually, that is expected because this model has much more complex conditions for node coverage (8) and distance constraint (9).

Because CPLEX did not solve all given instances, it was necessary to adopt PSO for solving this problem. We modified PSO method as follows:

- Each particle represents chosen positions for facilities in set of nodes.
- Distance constraint has been checked after each iteration, and if there is a pair of facilities on a distance less than allowed, the facility which covers more locations is removed.
- If a particle contains more (less) than P facilities, excess (lack) of facilities are removed (added) randomly.
- In the algorithm, swarm of 20 particles has been used. Stopping criteria for algorithm was 1000 iteration without finding better solution.
- Each instance has been solved 10 times with different random seeds and the best solution is chosen.
- Social and cognitive parameters from equation

(12) was obtained by numerical tests on instances with 300 nodes, 20 facilities, radius of coverage 3 and minimal allowed distance between facilities 4. Figure 2 presents obtained results and validity for choosing $c_2 = 0.3$ and $c_3 = 2.1$.

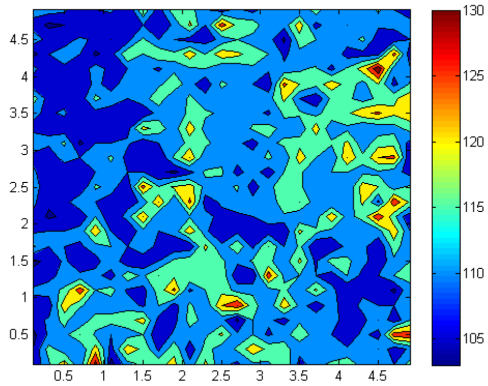

Fig. 2: Test results for determining social and cognitive parameters

Before solving instances with 500 nodes, it was necessary to show that developed algorithm finds all known results. Table 3 presents results of solving instances of 400 nodes by using PSO. It is obvious that PSO found all known optimal values, in many cases faster than CPLEX.

Table 2: Solving MinCLP-MC instances with 400 nodes by using PSO

n	***P***	***S***	Solution	Time (ms)
400	10	2	17	44253
400	10	3	43	50813
400	10	4	74	72396
400	10	5	82	66683
400	15	2	27	51231
400	15	3	73	51831
400	15	4	112	72446
400	15	5	124	69945
400	20	2	42	50693
400	20	3	104	91182
400	20	4	150	95172
400	20	5	167	90795

After showing proper work of developed algorithm, instances of 500 nodes have been solved. Because the optimal solutions for these instances are not known, quality of developed algorithm has been measured by average gap ($agap$) between obtained and best-known result and standard deviation σbetween these variables, by formulas:

$$agap = \frac{1}{N}\sum_{i=0}^{N}\frac{100Sol_i - BestSol}{BestSol}$$

and

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=0}^{N}(gap_i - agap)^2}$$

Parameter N represents a number of PSO runs on the same instance, and as mentioned before, in this study N=10.

Table 3: Solving MinCLP-MC instances with 500 nodes by using PSO

n	P	S	d	Solution	Solution time	Total time	$agap$	σ
500	10	2	4	21	73144	28369	0.000	0.000
500	10	3	4	59	82226	37279	0.677	0.830
500	10	4	4	89	110813	66221	1.123	1.004
500	10	5	4	107	131439	86497	3.831	3.195
500	15	2	4	36	77579	30560	0.277	0.833
500	15	3	4	97	90632	44321	0.618	0.505
500	15	4	4	137	99543	54519	1.313	1.072
500	15	5	4	154	115040	68813	4.090	3.168
500	20	2	4	55	75645	26443	0.181	0.545
500	20	3	4	135	116221	67388	1.333	0.645
500	20	4	4	184	213431	164569	2.554	1.667
500	20	5	4	211	137441	85605	2.559	2.163

Table 3 presents test results for solving MinCLP-MC instances with 500 nodes by using PSO. Obtained parameters (time, *agap* and standard deviation) show that PSO has successfully solved all given instances.

Figure 3 illustrates the graphical representation of solution MinCLP-MC instance with 300 nodes, 15 facilities, radius of coverage 4 and distance constraint 4.

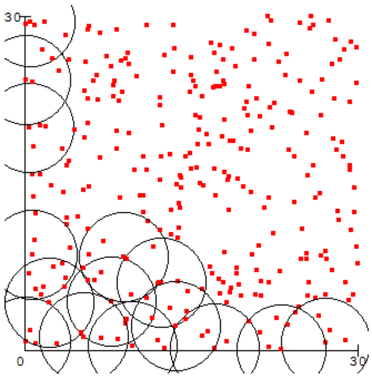

Fig. 3: Graphical representation of MinCLP-MC solution

As in the case of MinCLP-SC, all test results can be found on http://personal.ffuis.edu.ba/ddrakulic/phd/ /tests

5. Conclusion

In this paper we described two approaches to solving MinCLP-SC and MinCLP-MC by using CPLEX and PSO.

CPLEX is very powerful optimizer and it finds exact solutions, but in many cases it cannot find the optimal solution and it is necessary to solve the instance with another technique. We showed that CPLEX successful solve given instances, but it was unsuccessful in solving instances of MinCLP-MC with 500 nodes. Then, we described an adaptation of PSO method for solving MinCLP-MC and showed that implemented algorithm of PSO successfully solved all given instances.

In our previous research, we have shown that using fuzzy logic in modelling MCLP improves the quality of problem, but it also increase the computational complexity and exact methods can solve only instances with small dimension (see [7]). The main direction in future research will be related to defining and solving fuzzy minimal covering location problems by CPLEX and PSO.

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